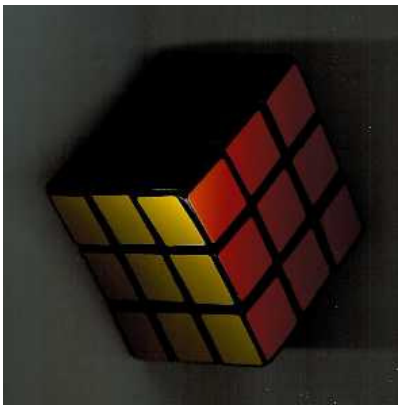
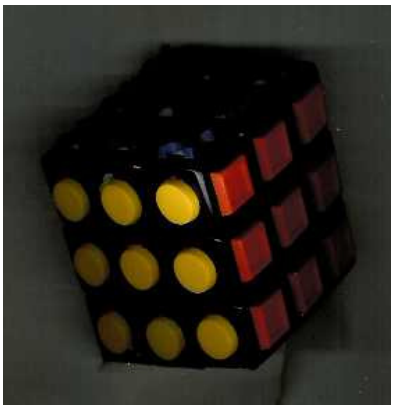


A Four-Move Solution to Rubik's Cube

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INTRODUCTION

This document describes a four-move solution for three versions of the cube puzzle originally sold by Ideal Toys under the name "Rubik's Cube".

One version of the puzzle is the original 3x3x3 cube which has colored squares on its sides. The colors are white, green, orange, blue, red or yellow. The second version is the 3x3x3 cube for the visually handicapped sold by LS&S Group. This version has raised symbols on its sides. The symbols are a triangle, a hollow circle, a hollow square, a filled circle, a filled square, or a letter "x". The third version is the 2x2x2 cube puzzle sold by Winning Moves, which looks like the head of Matt Groening's cartoon character Homer Simpson.

We use the 3x3x3 puzzle for the visually handicapped to describe the solution. The described solution carries over to the 3x3x3 original cube by substituting colors for symbols. The described solution carries over to the 2x2x2 cube by using only the moves for corner pieces.

THE 3x3x3 CUBE

Holding the cube straight up, we can name its sides as follows. There is U, the side which is up, D, the side which is down, L, the side to the left, R, the side to the right, F, the side in front, and B, the side in back. To repeat, the sides are U, up, D, down, L, left, R, right, F, front, and B, back.

We can also name three internal "wheels" of the cube. Wheel X, is the second horizontal row of the cube, which we can think of as a wheel that goes from the front, to the right, to the back, to the left side. Wheel Y, is the second column of the cube as viewed from the front, which we can think of as a wheel that goes from the front, to the up, to the back, to the down side. Wheel Z, is the second column of the cube as viewed from on the right, which we can think of as a wheel that goes from the right, to the up, to the left, to the down side.

Each side of the cube has a different symbol in its center position. We say that the cube is solved, when all 9 symbols on each side are the same.

Each side or wheel may be rotated either clockwise, or counter-clockwise. When we say "rotate", we mean "rotate by 90 degrees". We always specify whether to rotate clockwise, or counter-clockwise. "Clockwise" and "counter-clockwise" are defined from the point of view of the palm of your right hand, which is facing the side or wheel to be rotated.

In this solution, all rotations are done by the right hand. The left hand is used to hold those parts of the cube which must not be rotated.

To rotate a side or a wheel, do not first turn the cube to place that side or wheel in front of you. Instead, turn the right hand, so that its palm "faces" the side or wheel to be rotated. It is important to turn the whole cube only when the directions ask you to do so.

If the right hand is held, palm down, above the cube, its palm "faces" U. Palm up, below the cube, it faces D and X. Palm in front of the cube, it faces F. Palm at the back of the cube, it faces B. Palm to the right of the cube, it faces R and Y. In this description, the palm of the right hand never "faced" L or Z, but it doesn't matter: in this solution we never rotate L or Z.

To rotate a side, hold the wheel behind, and the opposite side of the cube, with the left hand. (The wheel "behind" and the opposite side may actually be closer to you). Then rotate the desired side in the desired direction with the right hand. To rotate a wheel, hold the side behind with the left hand, then rotate the wheel and the side in front with the right hand. Then hold the side behind and the rotated wheel with the left hand, and rotate back the side in front, again with the right hand.

Here are some examples of rotations. To rotate B, the back side, clockwise, hold wheel Z and side F from the front with the left hand, then rotate B clockwise from the back of the cube, with the right hand. (Rotate so the thumb goes up). To rotate X clockwise, hold U from above with the left hand, then rotate X and D clockwise from below with the right hand. (Rotate so the thumb goes to the right). Then hold U and X with the left hand and rotate D back counter-clockwise with the right hand. To rotate Y clockwise, hold L with the left hand, then rotate Y and R clockwise

from the right with the right hand (the thumb goes up), then hold L and Y with the left hand and rotate R back counter-clockwise with the right hand.

In this solution, rotations are indicated by letters. Clockwise rotations are indicated with capital letters, and counter-clockwise rotations are indicated with lower-case letters. For example,

r D R

means rotate the right side counter-clockwise, then rotate the down side clockwise, then rotate the right side clockwise.

If a rotation is to be repeated, we place a 2 after it. If a group of rotations is to be repeated, we surround the group with parentheses, and place a 2 after the parentheses. For example,

(y d2)2

means rotate the Y wheel counter-clockwise, then rotate the down side counter-clockwise twice, then rotate the Y wheel counter-clockwise again, then rotate the down side counter-clockwise twice again.

The 3x3x3 cube has 8 corner pieces, called "corners", 6 center pieces called "centers", and 10 pieces which are neither centers nor corners, called "edges". Corners have 3 visible surfaces, edges have 2 visible surfaces, and centers have one visible surface.

Please note that a piece may be in the correct position for the cube to be solved, without being in the correct orientation. We say that a piece is "done", when it is both in the correct position for a solution, and in the correct orientation.

THE PLAN

Our overall plan is the following. First, do the corners of U. Second, do the edges of U. Third, do the corners of D. Fourth, get the edges of D in the correct position. Fifth, get the edges of X in the correct position. Sixth, get the edges of D and X in the correct orientation. These six steps are the subject of the following six sections of this document.

While there are six steps in this solution, it is sufficient to know 4 "moves" or sequences of rotations. Move 1 interchanges corners. Move 2 re-orients corners in place. Move 3 interchanges edges. And move 4 re-orients edges in place.

Even though we use 4 moves, this document is organized in terms of the 6 steps, because the first side of the cube, e.g. the corners and edges of U, can be done readily without resort to memorized moves. In the "Discussion and Conclusion" section of this document, we review the 4 moves, and suggest ways to make each move easier to memorize.

Now, if you have not already done so, please scramble your cube. That is, make random rotations of its sides and wheels. Then pick a symbol for the center position of the top side of your solution. We assume that you choose the triangle. Turn the cube so the side which has the triangle at its center is at the top of the cube.

STEP 1. THE CORNERS OF U

The reader can discover how to do the corners of U without help, but here is one approach. Work on the corner at the right front of U. Start by naming the needed corner, as follows. With the left thumb, feel the symbol at the center of F. Suppose it's a hollow circle. With the left index finger, feel the symbol at the center of U. It's a triangle. With the right index finger, feel the symbol at the center of R. Suppose it's a hollow square. This means that the corner needed at the right front of U is named "hollow circle, triangle, hollow square".

Having named the corner needed at the right front of U, locate the needed corner without turning the cube. (If you cheat, don't forget to turn the cube back). Then, rotate sides to bring the needed corner to the right front of U, in the correct orientation. You may have to temporarily dislocate corners of U previously done, make the final rotation of the needed corner, then un-do the dislocation of the corners previously done. Then, turn the cube, so the current R becomes the new F. Repeat this paragraph for each corner of U.

STEP 2. THE EDGES OF U

The reader can also discover how to do the edges of U without help. If help is preferred, one way to proceed is the following.

Put the left thumb on the center of F and the left index finger on the center of U. Name the edge needed from F to U (name the center of F, then the center of U). Locate the needed edge without turning the cube. (Again, if you cheat, don't forget to turn the cube back). Rotate the needed edge to the bottom of L or R. It is better to leave the triangle facing out, not down.

Rotate Y counter-clockwise. This puts "the place" where the needed edge goes at the center bottom of F. Rotate the needed edge from the bottom of L or R (wherever you put it) into its "place". Rotate Y clockwise, so that the needed edge is in place back up at the edge from F to U. Un-do any dislocations that positioning the needed edge may have caused.

If the edge at F U is in the wrong orientation, you can rotate Y counter-clockwise, rotate D clockwise twice, rotate Y clockwise, and rotate D so that the needed edge is now at the bottom of L or R with the triangle facing out. Then, go back to the preceding paragraph.

Now turn the cube so R becomes F. Repeat the last 3 paragraphs for each edge of U.

STEP 3. THE CORNERS OF D

It is difficult if not impossible for the untrained reader to discover how to make any further progress toward a solution. (Enno Rubik had to be shown how to solve the cube). So in this section and in the next 3 sections we give 4 moves that are sufficient to solve the cube.

We begin with the corners of D. One way to do the corners of D is as follows. Name the corner needed at the front right of D. Rotate D until the needed corner is in the front right position, not necessarily in the correct orientation.

Turn the cube so that R becomes F. Name the corner now needed at the front right of D. If the named corner is already at the front right of D, go to the next paragraph. Otherwise, turn the cube so that R becomes F again, either once or twice, whichever it takes, to put the named corner at the front right of D. Execute the following move sequence to interchange the corners at the front right of D and the front left of D,

r D R D F d f ,

then turn the cube so F goes back to R. If the cube was turned twice, repeat the previous sentence.

Repeat the preceding paragraph until all four corners of D are in the correct position, not necessarily in the correct orientation.

Next, we get the four corners of D in the correct orientation. Tip the cube back, so that your original U (the side with the triangle at the center), becomes B.

Now, if necessary, turn the cube so R becomes U until the corners at the front left of U and the front right of U are both in the wrong orientation. If this is not possible, then turn R to U until just the corner in the front right of U is in the wrong orientation. If this too is not possible, the corners of your original D are done, so go to step 4, below. Then, execute the following sequence to rotate simultaneously the front left corner of U counter-clockwise, and the front right corner of U clockwise:

R B2 r u B2 U F u B2 U R B2 r f .

Repeat this whole paragraph until all of the corners of F (your original D) are in the correct position and in the correct orientation. (Think a little: can you avoid 2 applications on the same corners?)

STEP 4. THE EDGES OF D

At this point, your original U is still B, i.e. the symbol at the center of your original U is still the symbol at the center of B.

Turn the cube so U becomes L until one of the 2 surfaces on the edge from U to L has the same symbol as the center of your current F.

If no edge from U to L comes along with the same symbol as your current F then one of two situations has arisen. Either 1) all of the edges of your original D (now F) are in the correct position (not necessarily in the correct orientation), or 2) at least two edges of your original D are not in the correct position:

> If 1) all edges of your original D are in the correct position then go to STEP 5 below. If 2) at least two of the edges of your original D are not in the correct position, rotate F until one of those incorrect edges becomes edge F U, then perform the preparatory sequence, the 3-edge loop and the preparatory sequence un-do of the second following paragraph ("Next, ..."), and then return to the second preceding paragraph ("Turn the cube so U becomes L …"). You will have moved an incorrectly positioned edge out of your original D.

Otherwise, the edge from U to L now has the same symbol as the center of your current F. Note the symbol on the surface of the edge from U to L which is *not* the same as the symbol at the center of F. Rotate F until the upper left and right corners of F have the same symbol facing up as this other symbol on the edge from U to L.

Next, execute the preparing sequence

y d .

Then turn the cube so L becomes F. Now move 3 edges in a loop, U F to D F, F D to B U, and B U to F U, by executing the sequence

(y d2)2 y2 .

Now un-do the preparing sequence by turning F back to L and executing

D Y .

The edge that was formerly at U to L should now be at F to U. By adding the preparing and un-preparing sequences, we actually just moved U L to F U, U F to R U and U R to U L, with the same 3-edge interchanging move that we'll use later.

Repeat these paragraphs starting back at the one that begins "Turn the cube so U becomes L" until all the edges of F are in the correct position, not necessarily in the correct orientation. Finally, if necessary, rotate F until the symbol at the front corners of U is the same as the symbol at the center of U.

STEP 5. THE EDGES OF X

At this point, your original U is still B. Turn the cube so L becomes F.

Turn B to U until edge B U has the same symbols as the centers of U and F and edge U F has the same symbols as the centers of F and D. If edge B U never has the same symbols as the centers of U and F, turn the cube so L goes to U and then to R, and go back to the previous sentence ("Turn B to U …"). If edge B U still never has the same symbols as the centers of U and F, the edges of X are in the correct position, so go to section 6, below.

Otherwise, edge B U has the same symbols as the centers of U and F, and edge U F has the same symbols as the centers of F and D. Now, do our old friend "move 3 edges in a loop" again, U F to D F, F D to B U, and B U to F U, by executing the sequence

(y d2)2 y2 .

At this point, your original U is now L or R and all the edges of your original X should be in the correct position, but not necessarily in the correct orientation.

STEP 6. THE EDGES OF X AND D

If there are any edges in the wrong orientation, they are edges of your original X or D. Turn the cube so that the current edge U F is one of the edges which is in the wrong orientation.

If edge U R is also in the wrong orientation, go to the next paragraph. Otherwise, dislocate your partial solution, or turn the cube again (here it's not cheating), so that both edges U F and U R are in the wrong orientation. (Yes, edges of X and D in the wrong orientation always come in pairs).

Reorient edges U F and U R in place by executing the following sequence:

F X2 F2 X F U f x f2 x2 f u .

If you dislocated your solution, un-do the dislocation. If there are no more edges in the wrong orientation, the cube is solved. Otherwise, repeat the steps in this section.

DISCUSSION AND CONCLUSION

This solution is not one of the fastest, but it uses the smallest number of moves sufficient to separately interchange and re-orient corners and edges, namely 4. We repeat the four moves:

Move 1 interchanges the left and right front corners of D :

r D R D F d f .

Move 2 re-orients the left and right front corners of U:

R B2 r u B2 U F u B2 U R B2 r f .

Move 3 interchanges edges B U, U F, and F D:

(y d2)2 y2 .

And move 4 re-orients edges U F and U R:

F X2 F2 X F U f x f2 x2 f u .

Each of these four moves can be made easier to memorize, by speaking clockwise moves on a high pitch, and counter-clockwise moves on a low pitch.

Also, the two re-orienting move sequences can be spoken as syllable sequences. The move sequence to re-orient the corners may be said

ARE brew. BUFF you. BOO-ERB are eff .

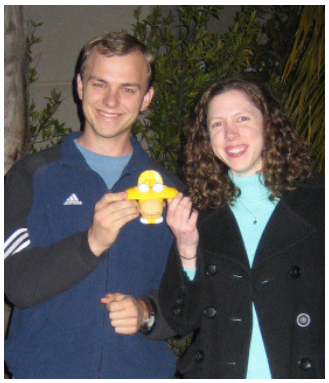
But remember that all the B's are repeated. The move sequence to re-orient the edges may be said

FIX FIX FOO , fix fix foo .

Remember that the "FIX FIX FOO" is all clockwise, and the "fix fix foo" is all counter-clockwise. Also remember that the middle 2 moves of "FIX FIX" are repeated, while the last 2 moves of "fix fix" are repeated.

To apply this solution to the 2x2x2 cube that looks like Homer Simpson, simply omit moves 3 and 4, since Homer has no edges.

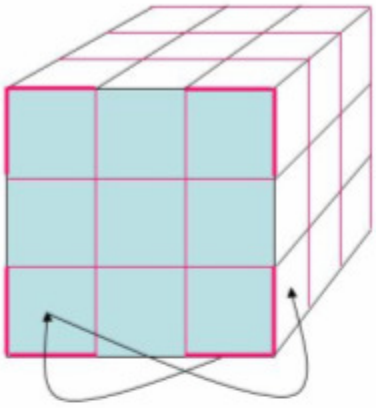
While this solution is not the fastest, with practice it can be used to solve a cube in less than 5 minutes. My thanks to Paolo Ienne, Michael Rael, Jenn Swiatek, Laura Marthaler and Sam Rodriguez for testing an earlier description. Comments welcome. Good luck!



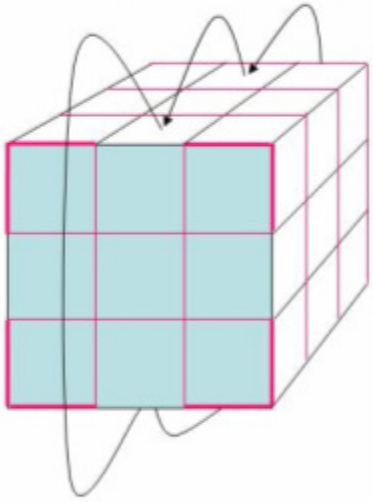
MR and JS hard at shirk!

26feb11: Appendex A. All four moves.

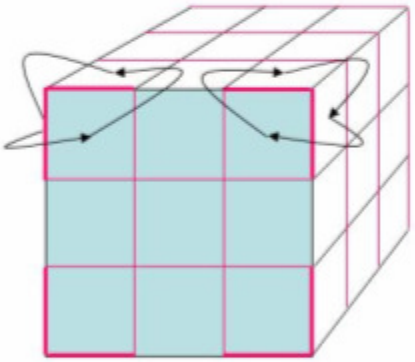
Interchange corners: rDRDFdf



Interchange edges: $yD^2yD^2y^2$



Rotate corners: $RB^2ruB^2UFuB^2URB^2rf$



Rotate edges: $FX^2F^2XFUfxf^2x^2fu$